Applied Math-I

Paper Code - 170012

Section A.

Q-1

(a) $\frac{\sqrt{-25}}{\sqrt{9}} = \frac{\sqrt{-1} \times \sqrt{25}}{\sqrt{9}} = \frac{i5}{3}$

(b) $z = -2+3i$

$|z| = \sqrt{(-2)^2+(3)^2} = \sqrt{4+9} = \sqrt{13}$

$\bar{z} = -2-3i$

(c) $\log_{10}(x+2) = 1$

or $10^1 = x+2$ (Exponential form)

$x = 10-2 = 8$

(d) Let $\log_4 256 = x$

or $256 = 4^x$ [Exponential form]

$4^4 = 4^x$

$\Rightarrow x = 4$

(e) $3! + 5! = 3\times 2\times 1 + 5\times 4\times 3\times 2+1$

$= 6 + 120$

$= 126$

(f) $^{10}P_2 = \frac{10!}{(10-2)!} = \frac{10!}{8!} = \frac{10\times 9\times 8!}{8!} = 90$

(g) The number of terms in expansion of $(x+2)^{-7}$ is $\underline{\infty}$ (Infinite)

(h) Minor of a_{22} in $\begin{array}{cc} & C_2 \\ & \begin{vmatrix} 3 & -4 \\ 7 & 10 \end{vmatrix} \\ R_2 \rightarrow & \end{array}$ is 3

Minor of a_{21} in $\begin{array}{cc} & C_1 \\ & \begin{vmatrix} 3 & -4 \\ 7 & 10 \end{vmatrix} \\ R_2 \rightarrow & \end{array}$ is -4

(i) <u>Zero Matrix</u> :- A matrix whose all elements are zero. e.g.

$$A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

(j)
$$A = \begin{bmatrix} 1 & 4 & 5 & 3 \\ 1 & 9 & -1 & 4 \end{bmatrix}$$

$$A' = \begin{bmatrix} 1 & 1 \\ 4 & 9 \\ 5 & -1 \\ 3 & 4 \end{bmatrix}$$

(k)
$$\cos 50 \cos 10 - \sin 50 \sin 10 = \cos(50+10)$$
$$= \cos 60$$
$$= \frac{1}{2}.$$

(l)
$$\sin 14\theta + \sin 2\theta$$
$$= 2 \sin \frac{14\theta + 2\theta}{2} \times \cos \frac{14\theta - 2\theta}{2}$$

$$= 2\sin\frac{16\theta}{2}\cos\frac{12\theta}{2}$$

$$= 2\sin 8\theta \cos 6\theta.$$

(m) We know that

$$\frac{D}{90} = \frac{2R}{\pi} = \frac{G}{100} = 1 \text{ right angle}.$$

Given $R = \frac{3\pi}{2}$.

So

$$\frac{D}{90} = \frac{\cancel{2} \times \frac{3\pi}{\cancel{2}}}{\cancel{\pi}}$$

$$D = 3 \times 90 = 270$$

(n) $(-1, 1)$

$x = -1 \qquad y = 1$

$$r = \sqrt{x^2+y^2} = \sqrt{(-1)^2+(1)^2} = \sqrt{2}$$

θ lies in II quadrant (as $x \to -ve$, $y \to +ve$)

So $\tan\theta = \frac{y}{x} = \frac{1}{-1} = -\tan 45$

$$\tan\theta = \tan(180-45)$$

$$= \tan 135^\circ$$

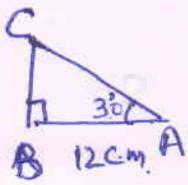
$$\theta = 135^\circ$$

(o) Slope of line $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

Given $x_1 = 7 \quad x_2 = 3$
$y_1 = 5 \quad y_2 = -1$

$$m = \frac{-1-5}{3-7} = \frac{-6}{-4} = \frac{3}{2}$$

(P) There must right angled triangle ABC



$\angle A = 30$, $AB = 12\,cm.$

In right angled triangle ABC

$$\tan 30 = \frac{CB}{BA}$$

$$\frac{1}{\sqrt{3}} = \frac{CB}{12}$$

or $BC = \dfrac{12}{\sqrt{3}}$

(q) Equation of Circle when end points of diameter are (x_1, y_1) & (x_2, y_2) is

$$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$$

(r) $x^2 + y^2 - 3x - 7 = 0$

Compare $x^2 + y^2 + 2gx + 2fy + c = 0$

$2g = -3 \qquad 2f = 0 \qquad c = -7$

$g = -\frac{3}{2} \qquad f = 0$

So Center $(-g, -f)$ is $\left(\frac{3}{2}, 0\right)$

Section B

Q-2

(i) $z_1 = 2-3i \quad z_2 = 4-5i$

$$\frac{z_1}{z_2} = \frac{2-3i}{4-5i} \times \frac{4+5i}{4+5i}$$

$$= \frac{8+10i-12i-15i^2}{4^2-(5i)^2}$$

$$= \frac{8-2i+15}{16+25} = \frac{23-2i}{41}$$

Real part $= \frac{23}{41}$ Imaginary part $= -\frac{2}{41}$

(ii) $(2x-3)+(y+3)i = 5-3i$

$\Rightarrow$ Compare Real & imaginary part

$2x-3 = 5$, $\quad y+3 = -3$

$2x = 8 \quad y = -6$

$x = 4$

(iii) $\log\frac{11}{5} + \log\frac{14}{3} - \log\frac{22}{15} = \log 7$

$$L.H.S = \log\left(\frac{11}{5}\times\frac{14}{3}\right) - \log\frac{22}{15}$$

$$= \log\frac{154}{15} - \log\frac{22}{15}$$

$$= \log\frac{\frac{154}{15}}{\frac{22}{15}} = \log\frac{154}{22} = \log 7$$

$= R.H.S$

(iv) $\lfloor n+1 = 12 \lfloor n-1$

$(n+1)\, n \cancel{\lfloor (n-1)} = 12 \cancel{\lfloor n-1}$

$n^2 + n - 12 = 0$

$n = -4,\ 3$

(v) $\left(3a + \frac{1}{2a}\right)^7$

$T_5 = ?$

$T_{R+1} = {}^N C_R\, x^{N-R}\, y^R$

$R = 4 \quad N = 7 \quad x = 3a \quad y = \frac{1}{2a}$

$T_{4+1} = {}^7 C_4\, (3a)^{7-4} \left(\frac{1}{2a}\right)^4$

$= \frac{7!}{3!\, 4!}\ 27a^3 \times \frac{1}{16a^4}$

$= \frac{7 \times 5 \times \cancel{6} \times \cancel{4!}}{\cancel{3 \times 2} \times 1 \times \cancel{4!}} \times \frac{27}{16}\ \frac{1}{a}$

$= \frac{945}{16a}$

(vi) $3\begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} x & 6 \\ -1 & -2w \end{bmatrix} + \begin{bmatrix} 4 & x+y \\ 2+w & 3 \end{bmatrix}$

$\begin{bmatrix} 3x & 3y \\ 3z & 3w \end{bmatrix} = \begin{bmatrix} x+4 & 6+x+y \\ -1+2+w & -2w+3 \end{bmatrix}$

$3x = x+4$, $3y = 6+x+y$

$2x = 4$, $2y = 6+x$

$x = 2$, $2y = 6+2 = 8$

$y = 4$

$3z = -1+z+w$, $3w = -2w+3$

$2z = -1+w$, $5w = 3$

$w = 3/5$

$2z = -1+\frac{3}{5} = -\frac{2}{5}$

$z = -\frac{1}{5}$

(vii)

$$A = \begin{bmatrix} 4 & 8 \\ 7 & 1 \end{bmatrix} \qquad |A| = 4-56 = -52$$

Minor of $a_{11} = 1$, Cofactor of $a_{11} = 1$

$a_{12} = 7$, $a_{12} = -7$

$a_{21} = 8$, $a_{21} = -8$

$a_{22} = 4$, $a_{22} = 4$

$$\text{adj} A = \begin{bmatrix} 1 & -7 \\ -8 & 4 \end{bmatrix}' = \begin{bmatrix} 1 & -8 \\ -7 & 4 \end{bmatrix}$$

$$A^{-1} = \frac{\text{adj} A}{|A|}, \quad |A| \neq 0.$$

$$A^{-1} = \frac{1}{-52}\begin{bmatrix} 1 & -8 \\ -7 & 4 \end{bmatrix} = \begin{bmatrix} -\frac{1}{52} & 8/52 \\ 7/52 & \frac{4}{52} \end{bmatrix}$$

viii) $\tan 70 = 2\tan 50 + \tan 20^\circ$

$$\tan 70 = \tan(50+20)$$

$$\frac{\tan 70}{1} = \frac{\tan 50 + \tan 20}{1 - \tan 50 \tan 20}$$

$$\tan 70(1 - \tan 50 \tan 20) = \tan 50 + \tan 20$$

$$\tan 70 - \tan 70 \tan 50 \tan 20 = \tan 50 + \tan 20$$

$$\tan 70 - \cancel{\cot 20} \tan 50 \cancel{\tan 20} = \tan 50 + \tan 20$$

as $\tan 70 = \tan(90-20)$
$= \cot 20$

$$\tan 70 - \tan 50 = \tan 50 + \tan 20$$

$$\tan 70 = 2\tan 50 + \tan 20$$

(ix) Wrong statement — so after Correction.

$$\sqrt{3}\cos 23^\circ - \sin 23 = 2\cos 53^\circ$$

R.H.S $2\cos 53 = 2\cos(30+23)$

$$= 2[\cos 30^\circ \cos 23^\circ - \sin 30^\circ \sin 23^\circ]$$

$$= 2\left[\frac{\sqrt{3}}{2}\cos 23^\circ - \frac{1}{2}\sin 23^\circ\right]$$

$$= \sqrt{3}\cos 23^\circ - \sin 23^\circ$$

$$= \text{L.H.S}$$

(x) Prove that $\frac{\sin A + \sin 3A}{\cos A + \cos 3A} = \tan 2A$

$$\frac{\cancel{2}\sin\frac{A+3A}{2}\cdot\cos\frac{A-3A}{2}}{\cancel{2}\cos\frac{A+3A}{2}\cos\frac{A-3A}{2}}$$

$$\frac{\sin 2A\ \cancel{\cos(-A)}}{\cos 2A\ \cancel{\cos(-A)}} = \tan 2A.$$

(xi)

$$\sin 2A = 2\sin A\cos A.$$

Given $\cos A = 5/13$

$$\sin A = \sqrt{1-\cos^2 A} = \sqrt{1-\frac{25}{169}} = \frac{12}{13}$$

$$\sin 2A = 2\times\frac{12}{13}\times\frac{5}{13} = \frac{120}{169}$$

(xii)

Two point form of line is

$$y-y_1 = \frac{y_2-y_1}{x_2-x_1}(x-x_1)$$

$x_1 = 7$, $y_1 = -3$, $x_2 = 5$, $y_2 = 1$

$$y+3 = \frac{1+3}{5-7}(x-7)$$

$$y+3 = \frac{4}{-2}(x-7)$$

$$y+3 = -2(x-7) \Rightarrow y+2x-11 = 0$$

(xiii)

$$4x - 7y = 11 \quad \text{—} (1)$$
$$5x + 2y = 7 \quad \text{—} (2)$$

$5 \times (4x - 7y = 11)$
$4 \times (5x + 2y = 7)$

$$\cancel{20x} - 35y = 55$$
$$\underline{\overset{-}{\cancel{20x}} \overset{-}{+} 8y = \overset{-}{28}}$$
$$-43y = 27$$
$$y = -\frac{27}{43}$$

Put $y = -\frac{27}{43}$ in (1)

$$4x - 7 \times -\frac{27}{43} = 11$$

$$4x + \frac{189}{43} = 11$$

$$4x = 11 - \frac{189}{43} = \frac{473 - 189}{43} = \frac{284}{43}$$

$$4x = \frac{284}{43}$$

$$x = \frac{71}{43}$$

Point of intersection $\left(\frac{71}{43}, \frac{-27}{43}\right)$

(xiv) Equation of circle

$$(x-h)^2 + (y-k)^2 = r^2 \quad — (1)$$

Given $(h, k) \to (2, 5)$

$$(x-2)^2 + (y-5)^2 = r^2 \quad — (2)$$

(2) passing through $(1,3)$ so put $x=1$, $y=3$ in equation (2)

$$(1-2)^2 + (3-5)^2 = r^2$$

$$1 + 4 = r^2$$

$$5 = r^2$$

Equation (1) becomes

$$(x-2)^2 + (y-5)^2 = 5$$

(xv) $2x^2 + 2y^2 - 6x + 8y - 1 = 0$

$$x^2 + y^2 - 3x + 4y - \tfrac{1}{2} = 0 \quad — (1)$$

Compare $x^2 + y^2 + 2gx + 2fy + c = 0$

$$2g = -3, \quad 2f = 4, \quad c = -\tfrac{1}{2}$$

$$g = -\tfrac{3}{2} \qquad f = 2$$

$$\text{radius} = \sqrt{g^2 + f^2 - c}$$

$$= \sqrt{\frac{9}{4} + 4 + \frac{1}{2}} = \sqrt{\frac{9+16+2}{4}}$$

$$= \sqrt{\frac{27}{4}}$$

Section C

Q-3

$$A = \begin{bmatrix} 4 & 5 \\ -1 & 2 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix}$$

C matrix not given Completely

So taking $C = \begin{bmatrix} 5 & 0 \\ 4 & 0 \end{bmatrix}$.

$$A + 2B - 3C = \begin{bmatrix} 4 & 5 \\ -1 & 2 \end{bmatrix} + 2\begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix} - 3\begin{bmatrix} 5 & 0 \\ 4 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 5 \\ -1 & 2 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 4 & 6 \end{bmatrix} - \begin{bmatrix} 15 & 0 \\ 12 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 0 \\ 3 & 8 \end{bmatrix} - \begin{bmatrix} 15 & 0 \\ 12 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -9 & 0 \\ -9 & 8 \end{bmatrix}$$

Q 4

$$\frac{(1-3x)^{1/2} + (1-x)^{5/3}}{\sqrt{4-x}} \simeq a + bx$$

We know

$$(1+x)^N = 1 + Nx$$

$$(1-3x)^{1/2} = 1 + \frac{1}{2}(-3x) = 1 - \frac{3x}{2} \qquad -①$$

$$(1-x)^{5/3} = 1 + \frac{5}{3}(-x) = 1 - \frac{5x}{3}$$

Now $$(1-3x)^{1/2} + (1-x)^{5/3} = 1 - \frac{3x}{2} + 1 - \frac{5x}{3}$$

$$= 2 - \frac{3x}{2} - \frac{5x}{3}$$

$$= \frac{12 - 9x - 10x}{6}$$

$$= \frac{12 - 19x}{6} = 2 - \frac{19x}{6}$$

L.H.S

$$\frac{(1-3x)^{1/2} + (1-x)^{5/3}}{\sqrt{4-x}} = \frac{\left(2 - \frac{19x}{6}\right)}{(4-x)^{1/2}}$$

$$= \left(2 - \frac{19x}{6}\right)(4-x)^{-1/2}$$

$$= \left(2 - \frac{19x}{6}\right)4^{-1/2}\left(1 - \frac{x}{4}\right)^{-1/2}$$

$$= \frac{1}{2}\left(2 - \frac{19x}{6}\right)\left(1 + \left(-\frac{1}{2}\right)\left(-\frac{x}{4}\right)\right)$$

$$= \frac{1}{2}\left(2 - \frac{19x}{6}\right)\left(1 + \frac{x}{8}\right)$$

$$= \frac{1}{2}\left(2 + \frac{x}{4} - \frac{19x}{6}\right)$$

$$= \frac{1}{2}\left(\frac{48 + 6x - 76x}{24}\right)$$

$$= \frac{1}{2}\left(\frac{48 - 70x}{24}\right)$$

$$= 1 - \frac{70}{48}x$$

$$= 1 - \frac{35}{24}x$$

$$\Rightarrow \quad a = 1 \qquad b = -\frac{35}{24}$$

Q-5. $\cos 6^\circ \cos 42^\circ \cos 66^\circ \cos 78^\circ = \frac{1}{16}$

L.H.S.

$$\frac{1}{2}\left(2\cos 6^\circ \cos 66^\circ\right) \frac{1}{2}\left(2\cos 42^\circ \cos 78^\circ\right)$$

$$\frac{1}{4}\left[\cos(6+66^\circ) + \cos(6-66^\circ)\right]\left[\cos(42^\circ+78^\circ) + \cos(42^\circ-78^\circ)\right]$$

$$\frac{1}{4}\left[\cos 72^\circ + \cos(-60^\circ)\right]\left[\cos 120^\circ + \cos(36^\circ)\right]$$

$$\frac{1}{4}\left[\cos 72 + \frac{1}{2}\right]\left[-\frac{1}{2} + \cos 36\right] \quad — ①$$

as $\cos 72^\circ = \cos(90^\circ - 18^\circ) = \sin 18^\circ = \frac{\sqrt{5}-1}{4}$

and $\cos 36^\circ = \frac{\sqrt{5}+1}{4}$

So ① becomes

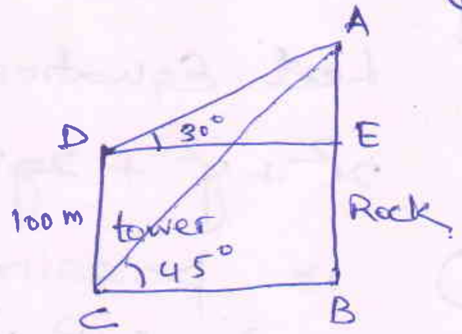
$$\frac{1}{4}\left[\frac{\sqrt{5}-1}{4} + \frac{1}{2}\right]\left[-\frac{1}{2} + \frac{\sqrt{5}+1}{4}\right]$$

$$\frac{1}{4}\left[\frac{\sqrt{5}-1+2}{4}\right]\left[\frac{-2+\sqrt{5}+1}{4}\right]$$

$$\frac{1}{64}(\sqrt{5}+1)(\sqrt{5}-1) = \frac{(\sqrt{5})^2-1}{64} = \frac{4}{64} = \frac{1}{16}$$

$$= \text{R.H.S}$$

Q-6



In right angle triangle AED

$$\tan 30 = \frac{AE}{DE}$$

$$\frac{1}{\sqrt{3}} = \frac{AE}{DE}$$

$$\boxed{DE = \sqrt{3}\,AE} \quad \text{— (1)}$$

In right angle triangle ABC

$$\tan 45^\circ = \frac{AE+100}{BC}$$

$$1 = \frac{AE+100}{DE} \qquad \text{as } BC = DE$$

$$DE = AE + 100. \quad \text{— (2)}$$

Using (1) in (2)

$$\sqrt{3}\,AE = AE + 100$$

$$(\sqrt{3}-1)\,AE = 100$$

$$AE = \frac{100}{\sqrt{3}-1}$$

So height of Rock is $AB = 100 + AE$

$$= 100 + \frac{100}{\sqrt{3}-1}$$

$$= \frac{100(\sqrt{3}-1) + 100}{\sqrt{3}-1}$$

$$= \frac{100\sqrt{3}}{\sqrt{3}-1}$$

Q.7.

Let Equation of Circle is

$x^2+y^2+2gx+2fy+c=0$ — ①

{taking $(2,0)$ as $(2,0)$ in this Question}

① is passing through $(-1,-1)$, so

$(-1)^2+(-1)^2+2g(-1)+2f(-1)+c=0$

$-2g-2f+c+2=0$ — ②

① passing through $(2,0)$ so

$2^2+0+2g\times 2+2f(0)+c=0$

$4g+c+4=0$ — ③

① passing through $(4,2)$, so

$4^2+2^2+2g\times 4+2f\times 2+c=0$

$8g+4f+c+20=0$ ④

To eliminate c

③ − ②

$6g+2f+2=0$ or $3g+f+1=0$ — ⑤

Again. ④ − ③

$4g+4f+16=0$ or $g+f+4=0$ ⑥

Solving ⑤ & ⑥

$$\begin{array}{r} 3g+\cancel{f}+1=0 \\ -\;g\;\underline{\pm}\cancel{f}\;\underline{\pm 4}=0 \\ \hline 2g\quad -3=0 \end{array}$$

or $g=\frac{3}{2}$

Put $g=\frac{3}{2}$ in ⑤

$3g+f+1=0$

$3\times\frac{3}{2}+f+1=0$ $\quad f=-\frac{11}{2}$

Put $g = 3/2$, $f = -11/2$ in Equation 3.

$$4g + c + 4 = 0$$

$$\cancel{4} 2 \times \frac{3}{\cancel{2}} + c + 4 = 0$$

$$c = -10.$$

Hence Equation of Circle is

$$x^2 + y^2 + 2 \times \frac{3}{2} x + 2\left(-\frac{11}{2}\right) y - 10 = 0$$

$$x^2 + y^2 + 3x - 11y - 10 = 0.$$